

Compound Interest with regular payments

Now suppose that we deposit \$1000, then pay \$100 per year into the fund.

Calculator, iterative formula: $1000 =$
 $ANS \times 1.1 + 100 =$ $=$ $=$

Google sheet, iterative formula:

Year	Value
0	1000
1	$= B2 \times 1.1 + 100$

How the math works out:

Year	Calculation
0	1000
1	$1000 \times 1.1 + 100$
2	$(1000 \times 1.1 + 100) \times 1.1 + 100 = 1000 \times 1.1^2 + 100 \times 1.1 + 100$
3	$1000 \times 1.1^3 + 100 \times 1.1^2 + 100 \times 1.1 + 100$
4	$1000 \times 1.1^4 + 100 \times 1.1^3 + 100 \times 1.1^2 + 100 \times 1.1 + 100$
5	$1000 \times 1.1^5 + 100 \times 1.1^4 + 100 \times 1.1^3 + 100 \times 1.1^2 + 100 \times 1.1 + 100$
	$= 1000 \times 1.1^5 + 100 (1.1^4 + 1.1^3 + 1.1^2 + 1.1^1 + 1)$
	<div style="border-top: 1px solid black; width: 50%; margin: 0 auto; padding-top: 5px;"> this is called a 'geometric sequence' </div>

TVM (time value money) parameters:

Client controls: Present Value
 Payment
 Periods

Bank controls: Rate of Interest
 Compounding Intervals

1. Determine the future value for each investment using a TVM calculator

- a) Monthly payments of \$200.00 for 50 years invested at 4.8% compounded monthly.

Present Value = 0

Payment = -200

Future Value = ?

Annual Rate (%) = 4.8

Periods = 600

Compounding = monthly

\$ 498 526.60

Invest: $200 \times 600 = 120000$

Interest: \$ 378 526.60

- b) Semi-annual payments of \$1750.00 for 20 years invested at 5.6% compounded semi-annually.

Present Value = 0

Payment = -1750

Future Value = ?

Annual Rate (%) = 5.6

Periods = 40

Compounding = semi-annual

\$ 126127.32

Invest: \$ 700,000

Interest: \$ 56,127

- c) Quarterly payments (every 3 months) of \$50.00 for 40 years invested at 8.4% compounded quarterly.

Present Value =

Payment =

Future Value =

Annual Rate (%) =

Periods = 160

Compounding =

\$ 63820.79

Invest: 8000

Interest: 55,820

(high rate,
long time)

- d) Semi-annual payments of \$5500.00 for 12 years invested at 6.5% compounded semi-annually.

Present Value =

Payment =

Future Value =

Annual Rate (%) =

Periods = 24

Compounding =

\$ 195 389.47

Invest: \$132000

Interest: \$63 389.47

2. Determine the unknown values.

- a) Monthly payments of \$100.00 for 6 years compounded monthly. The future value is \$7800.61. What is the interest rate?

Present Value = 0
Payment = -100
Future Value = 7800.61
Annual Rate (%) = ?
Periods = 72
Compounding = monthly

2.68%

- b) A 7 year investment at 3.5% compounded semi-annually. The future value is \$3927.38. What are the semi-annual payments?

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

\$250

- c) Quarterly payments of \$20,000 invested at 4.75% compounded quarterly. The future value is \$1,080,978.04. How many payments do you end up making? (How many periods?)

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

42 payments

3. Darlene has invested \$350 at the end of each month, at 7.2% compounded monthly, for 18 years.

a) What is the investment's future value?

Present Value = 0

Payment = -350

\$154,030.54

Future Value =

Annual Rate (%) = 7.2

Periods = 216

Compounding = monthly

b) How much interest has she earned?

$$216 \times 350 = 75,600$$

$$\text{Interest} = \$78,430$$

4. Fraser, who is currently 16 years old, wants to buy a car when he is 21. He deposits \$600 every 3 months (quarterly) into a savings account that earns 6.8%, compounded quarterly.

a) How much money will he have to buy his car when he turns 21? (That is in 5 years).

Present Value =

Payment =

\$14,150.77

Future Value =

Annual Rate (%) =

Periods = 20

Compounding =

b) How much interest will he have earned?

\$2,150.77

9. How long will it take for \$1000 payments every 6 months (semi-annually) to grow to more than \$10000 if the interest rate is 7.5%, compounded semi-annually?

Present Value =

Payment =

Future Value =

Annual Rate (%) =

Periods = 8.65

Compounding =

so, 9 payments.

1 payment every six months,
so 54 months = 48 + 6 months

4 years 6 months

4½ years.

Brain Teaser: Which of the following is the odd one out?

A. GOAT

B. BULL

C. CHICKEN

D. LION

E. RAM