5.7 – Investments Involving Regular Payments

Compound Interest Formula: $FV = P\left(1+\frac{r}{n}\right)^{(n\times t)}$

We have been using the compound interest formula to work out the final amounts for one-time investments. But realistically when we are saving money it will most likely be smaller payments into the account more often.

Example 1: An investment is made by depositing $500 every year into an RRSP (Registered Retirement Savings Plan) at 6% compounded annually for 20 years. What is the Final Value?

Method 1 on calculator

500 enter
×1.06+500 enter } 20 times.

$18392,80

Method 2:

$Y_1 = 500$
$Y_2 = 500 \times 1,06 + 500$
$Y_3 = (500\times1,06+500)\times1,06+500$
$= 500\times1,06^2 + 500\times1,06 + 500$

$Y_{20} = 500\times1,06^{19} + 500\times1,06^{18} + \dots + 500$
$= 500(1,06^{19} + 1,06^{18} + \dots + 1,06^0)$
$= \frac{500(1,06^{20}-1)}{1,06-1} = 18392,80$

GRADE 12 MATH

METHOD 3:

We can use "**The Value of Money**" (TVM) **Solver** on a financial app such as **EZ Financial Calculator**

Present Value (PV) is the initial balance of your investment account (usually 0)
Payment (PMT) is the amount of the regular payment into the account. (negative)
Future Value (FV) is how much you have in your account at the end of the term.
Annual **Rate** (%) is the interest rate left as a percent
Periods is how many payments you will be making into the account overall
Compounding how many times is your interest rate compounded per year?
Mode are you making payments at the beginning or end of a period?
(Leave this on End)
Decimal – there are two decimals when we write monetary amounts, so leave on two.

Note:

Money out of your pocket is negative.

Money going into your pocket is positive.

So for our question.....

Present Value = 0
Payment = −500
Future Value = ?
Annual Rate (%) = 6%
Periods = 20
Compounding = annually

$18392,80

When you are ready to Calculate your answer:

press the button of the item you want to solve (from the list on the right)

t=5

Example 2: Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually.

n=2

a) How much money will be in the account at the end of 5 years?

Present Value = 0
Payment = −500
Future Value = ?
Annual Rate (%) = 3.8
Periods = 10
Compounding = Semiannually

Periods = n × t (4 years)
= 2 × 5
(compounding) times per year

FV = $5449.90

b) How much of this money will be earned interest?

Interest = ~~Total Pay~~ FV − Total Payments
= 5449.90 − 10×500 = $449.90

Example 3: Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually for 5 years.

a) What is Adam's future value?

Present Value = 0
Payment = −200
Future Value = ?
Annual Rate (%) = 5
Periods = 5
Compounding = annually

~~FV $1051.27~~
FV = $1105.13

b) Blake made a single investment (use compound interest formula!) of $865 at 5%, compounded annually. At the end of 5 years what is his future value (final amount)?

$$865\left(1+\frac{0.05}{1}\right)^{5} = \$1103.98$$

c) Who earned more interest? Why?

Adam, interest = $1105.13 − $1000 = 105.13
Blake, interest = $1103.98 − 865 = more.

More money in the bank for a longer time.

n=4

Example 4: Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is $10 059.07. What annual rate of interest does Jeremiah's investment earn?

t=3

Present Value = 0
Payment = −750
Future Value = 10059.07
Annual Rate (%) =
Periods = n×t = 12
Compounding = quarterly

8%

Example 5: Celia wants to have $300,000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded monthly.

t

n=12

a) What regular payments must Celia make at the end of each month to meet her goal of $300,000?

Present Value = 0
Payment =
Future Value = 300 000
Annual Rate (%) = 10,8
Periods = 240
Compounding = monthly

Periods = n×t
= 12×20
= 240

Payment = $355,84

b) How much interest will she earn over the 20 years?

Interest = FV − Total payments
= 300000 − 240×355.84 = $214598.40

Example 6: On Luis's 20th birthday, he started making regular $1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have more than $18 000?

Present Value = 0
Payment = −1000
Future Value = 18000
Annual Rate (%) = 3,5
Periods = 2, 15.78
Compounding = semi-annually

We need 16 payments to have more than 18000

when periods = 16, FV = $18281.68
when periods = 15, FV = $16984.45

16 periods is 8 years.

Assignment: Worksheet 5G

Online TVM Solver: http://www.zenwealth.com/BusinessFinanceOnline/TVM/TVMCalculator.html

1. Determine the future value for each investment using a TVM calculator

a) Monthly payments of $200.00 for 50 years invested at 4.8% compounded monthly.

Present Value = 0
Payment = –200
Future Value = ?
Annual Rate (%) = 4.8
Periods = 600
Compounding = monthly

$498 526,60

b) Semi-annual payments of $1750.00 for 20 years invested at 5.6% compounded semi-annually.

Present Value = 0
Payment = –1750
Future Value = ?
Annual Rate (%) = 5.6
Periods = 40
Compounding = semi-annual

$126127,32

c) Quarterly payments (every 3 months) of $50.00 for 40 years (t) invested at 8.4% compounded quarterly. (n=4)

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods = n x t = 40 x 4 = 160
Compounding =

$63 820,79

d) Semi-annual payments of $5500.00 for 12 years (t) invested at 6.5% compounded semi-annually. (n=2)

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods = 24
Compounding =

$195389,47

2. Determine the unknown values.

a) Monthly payments of $100.00 for 6 years compounded monthly. The future value is $7800.61. What is the interest rate?

Present Value = 0
Payment = –100
Future Value = 7800.61
Annual Rate (%) = ?
Periods = 72
Compounding = monthly

2.675%

b) A 7 year investment at 3.5% compounded semi-annually. The future value is $3927.38. What are the semi-annual payments?

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

$250

c) Quarterly payments of $20,000 invested at 4.75% compounded quarterly. The future value is $1,080,978.04. How many payments do you end up making? (How many periods?)

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

42 periods
which is 10.5 years.

n=12 t=2

5. Zoey deposited the same amount of money at the end of each month for 2 years in a savings account that earned 6% interest, compounded monthly. She ended up with $5000. How much did Zoey deposit each month?

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods = 12x2=24
Compounding =

$196.60

6. Jane plans to retire in 35 years (when she is 55) and hopes to have $1,000,000 saved. For each investment option below, how much does she need to invest at the end of each month to reach her goal?

a) 11.6% compounded monthly

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

$173.07

b) 6.9% compounded monthly

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

$568.60

thats a big difference.

7. What interest rate, compounded monthly, is required to make monthly payments of $500 grow to $35000 in 5 years?

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

6.13%

8. Which investment will earn more interest? Show your calculations.

A. A one-time payment of $5000 invested at 6%, compounded annually, for 5 years.

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{(n \times t)}$

$6691.13

B. $1000 invested every year at 6%, compounded annually, for 5 years.
(Use your TVM calculator)

$5637.09

Present Value =
Payment =
Future Value =
Annual Rate (%) =
Periods =
Compounding =

Draw two graphs
y=5000 Compare on a spreadsheet.

3. Darlene has invested $350 at the end of each month, at 7.2% compounded monthly, for 18 years.

↑high ↑long

a) What is the investment's future value?

Present Value = 0
Payment = -350
Future Value = $154030.54
Annual Rate (%) = 7.2
Periods = 18x12 = 216
Compounding = monthly

b) How much interest has she earned?

Total payments $350 \times 216 = 75600$

Interest = ~~75600~~ $154030.54 - 75600$

$= \$78,430$

a lot. (More than 100% rate of return).

4. Fraser, who is currently 16 years old, wants to buy a car when he is 21. He deposits $600 every 3 months (quarterly) into a savings account that earns 6.8%, compounded quarterly.

a) How much money will he have to buy his car when he turns 21? (That is in 5 years).

Present Value = 0
Payment = -600
Future Value = $14150,77
Annual Rate (%) = 6.8
Periods = 5x4 = 20
Compounding = Quarterly

b) How much interest will he have earned?

Interest = $14150.77 - 20 \times 600$

$= 14150.77 - 12000 = \$2150.77$