

Savings Accounts, Simple and Compound Interest

For a 'high interest' savings account, where the owner can access the funds at any time, the interest rates tend to be between:

For a term investment, the owner must deposit the funds for a stated period, eg, 10 years. In that time the bank can use the money to invest. The bank can then offer a higher interest rate. The amount earned can be fixed (more security, lower interest) or left to the events of the market (it is possible to lose money over shorter periods of time, but left over longer periods of time the interest is generally higher).

Interest rates for investments can be anywhere between:

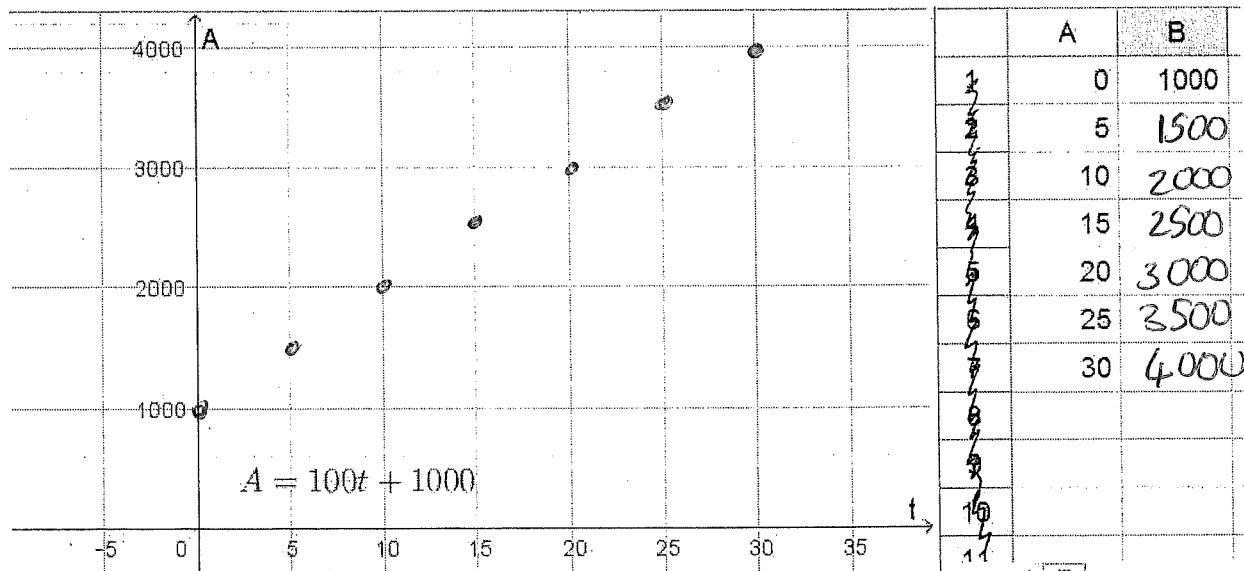
Simple Interest

The interest is calculated on the principle only. The interest is constant. The relationship between future value and time is linear:

Let the Present Value of an investment be \$1000.

Let the rate of interest be 10%.

A simple interest model plays out as follows:



A simple interest model is useful if there is some reason to remove interest each year. For example, a non-profit organization has a trust fund that takes care of a \$40 000,000 endowment. The interest on the endowment is 4%. They remove the interest each year to pay for operating costs. How much interest does the fund generate each year? How many salaries is that?

$40\,000\,000$
 1% is 400 000
 4% is 1 600 000

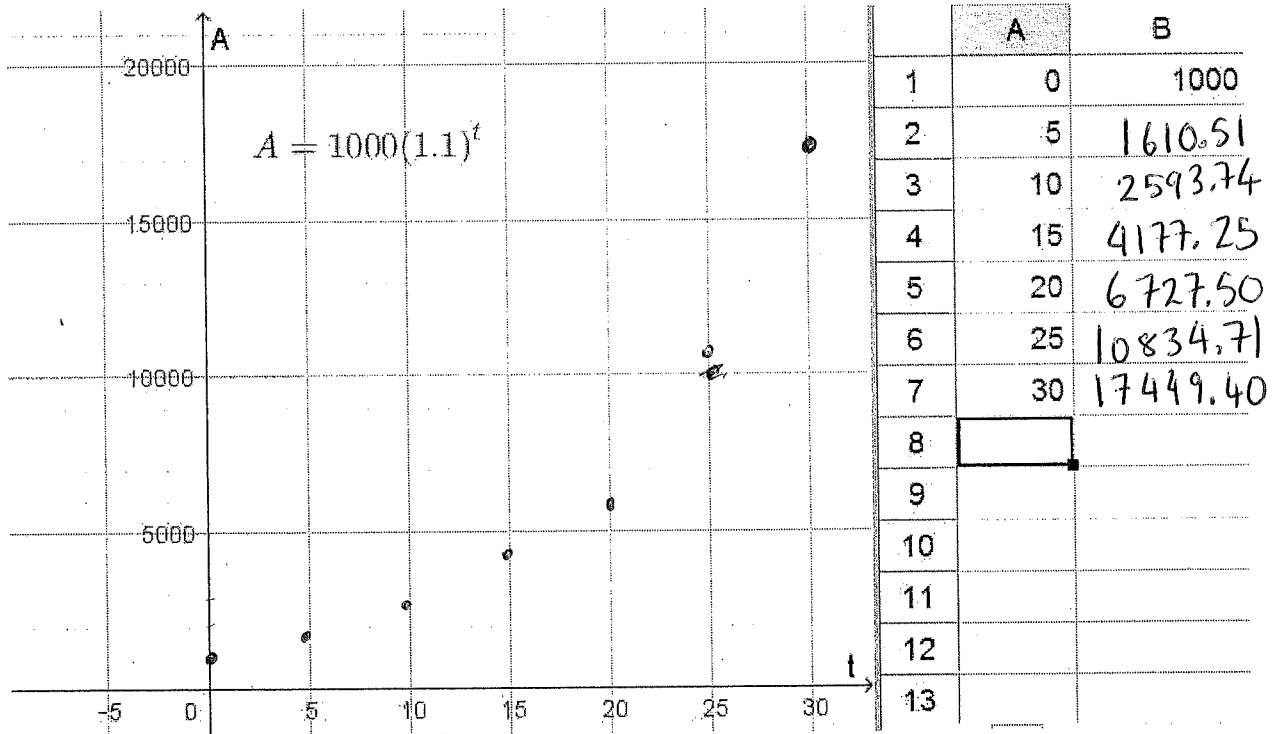
\$1,600 000

Suppose 50K salary average.
 $1600K \div 50K = 32$ employees.

Compound Interest

Usually, interest is compounded – interest earns interest.

Let's recalculate the table, and plot the new points.



Different Compounding Periods

Banks rarely add interest just once per year. The compounding period can be semi-annual, quarterly, monthly, weekly or daily.

Let's check out one year, $P = \$1000$, $r = 10\%$, with these compounding periods:

Compounding period	Times per year (n)	Rate \div n	r (decimal)	Calculation	Value at one year
Annual	1	$\frac{10}{1} = 10$	0.1	$1000(1+0.1)^1$	1100
Semi-annual	2	$\frac{10}{2} = 5$	0.05	$1000(1+0.05)^2$	1102.5
Quarterly	4	$\frac{10}{4} = 2.5$	0.025	$1000(1+0.025)^4$	1103.81
Monthly	12	$\frac{10}{12} = 0.8\bar{3}$	0.008 $\bar{3}$	$1000(1+0.008\bar{3})^{12}$	1104.71
Weekly	52	$\frac{10}{52} = \dots$	$\frac{10}{52} \div 100$	$1000(1+\frac{10}{5200})^{52}$	1105.06
Daily	365	$\frac{10}{365} = \dots$	$\frac{10}{365} \div 100$	$1000(1+\frac{10}{36500})^{365}$	1105.16

Example 1:

Invest \$3000, for 5 years at 4% interest compounded monthly.

$$FV = PV \left(1 + \frac{r}{n}\right)^{n \times t}$$

$$PV = 3000$$

$$r = 0,04$$

$$n = 12 \text{ (monthly)}$$

$$t = 5 \text{ years}$$

$$\left. \begin{array}{l} n \times t = 60 \end{array} \right\}$$

total compounding periods

$$FV = 3000 \left(1 + \frac{0,04}{12}\right)^{60}$$

$$= \$3662,99$$

Example 2:

Borrow \$5000 for 3 years at 7% interest compounded daily.

$$PV = 5000$$

$$r = 0,07$$

$$n = 365 \text{ (daily)}$$

$$t = 3 \text{ years}$$

$$\left. \begin{array}{l} n \times t = 1095 \end{array} \right\}$$

$$FV = 5000 \left(1 + \frac{0,07}{365}\right)^{1095}$$

$$= \$6168,27$$

Compound Interest Worksheets

Name _____

Worked Solutions.

Calculate the total amount of the investment or total paid in a loan in the following situations:

1.) Your 3 year investment of \$20,000 received 5.2% interest compounded semi annually. What is your total return?

$$\text{Answer: } 20000 \left(1 + \frac{0.052}{2}\right)^{2 \times 3} = 23329.97$$

2.) You borrowed \$59,000 for 2 years at 11% which was compounded annually. What total will you pay back?

$$\text{Answer: } 59000 \left(1 + \frac{0.11}{1}\right)^{2 \times 1} = 72693.90$$

3.) Your allowance of \$190 got 11% compounded monthly for 1 2/3 years. What's it worth after the 1 2/3 years? $\frac{2}{3}$ year = 8 months, $1\frac{2}{3}$ years = 20 months.

$$\text{Answer: } 190 \left(1 + \frac{0.11}{12}\right)^{20} = 228.04$$

4.) Your 6 1/4 year investment of \$40,000 at 14% compounded quarterly is worth how much now? $6\frac{1}{4}$ years = 25 quarters.

$$\text{Answer: } 40000 \left(1 + \frac{0.14}{4}\right)^{25} = 94529.80$$

5.) You borrowed \$1,690 for 5 1/2 years at 5.7% compounded semi annually. What total will you pay back? $5\frac{1}{2}$ years = 11 half years.

$$\text{Answer: } 1690 \left(1 + \frac{0.057}{2}\right)^{11} = 2302.15$$

6.) Your \$440 gets 5.8% compounded annually for 8 years. What will your \$440. be worth in 8 years?

$$\text{Answer: } 440 \left(1 + \frac{0.058}{1}\right)^8 = 690.78$$

7.) Your \$54,200 2 year car loan is at 15.1% compounded annually. What will you have paid for your car after 2 years?

$$\text{Answer: } 54200 \left(1 + \frac{0.151}{1}\right)^2 = 71804.21$$

8.) You invest \$55 at 10% compounded annually for 3 years. How much will your investment be worth in 3 years?

$$\text{Answer: } 55 \left(1 + \frac{0.1}{1}\right)^3 = 55 \times 1.1^3 = 73.21$$

9.) Your 8 year loan of \$12,200 is at 5.3% compounded annually. How much will you have paid in total for your loan?

$$\text{Answer: } 12200 \left(1 + \frac{0.053}{1}\right)^8 = 12200(1.053)^8 = 18441.10$$

10.) You invest \$1,900 at 4% and it's compounded semi annually for 3 years. How much will your \$1,900 be worth in 3 years?

$$\text{Answer: } 1900 \left(1 + \frac{0.04}{2}\right)^6 = 2139.71$$