PC 11 Solving Quadratic Equations

$$x^2 = 49$$

$\Rightarrow \quad x = \pm\sqrt{49}$

$\Rightarrow \quad x = -7, \; x = 7$

or: $x^2 - 49 = 0$

$\Rightarrow \quad (x-7)(x+7) = 0$

$\Rightarrow \quad x-7=0, \; x+7=0$

$\Rightarrow \quad x = 7, \; x = -7$

$$x^2 + 17x + 50 = 3x + 5$$

$\Rightarrow \quad x^2 + 14x + 45 = 0$

$\Rightarrow \quad (x+5)(x+9) = 0$

$\Rightarrow \quad x+5=0$ or $x+9=0$

$\Rightarrow \quad x=-5$ or $x=-9$

$$x^2 + 11x - 39 = x$$

$\Rightarrow \quad x^2 + 10x - 39 = 0$

$\Rightarrow \quad (x-3)(x+13) = 0$

$\Rightarrow \quad x-3=0$ or $x+13=0$

$\Rightarrow \quad x=3$ or $x=-13$

$$x^2 + 4x = 7$$

$\Rightarrow \quad x^2 + 4x - 7 = 0$

$(x\,?\,1)(x\,?\,7) = 0$

doesn't factor.

Use QF or complete square.

$a=1, \; b=4, \; c=-7$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16+28}}{2a}$$

$$= \frac{-4 \pm \sqrt{44}}{2}$$

$$= \frac{-4 \pm \sqrt{4 \times 11}}{2}$$

$$= \frac{-4 \pm 2\sqrt{11}}{2}$$

$= -2 \pm \sqrt{11}$ (exact values)

(or, $x = -1.32$
$x = 5.32$
correct to 2.d.p.)

$$\left[\ x = 10 - \frac{21}{x}\ \right] \qquad \times x$$

$$\Rightarrow \quad x^2 = 10x - 21$$

$$\Rightarrow \quad x^2 - 10x + 21 = 0$$

$$\Rightarrow \quad (x-3)(x-7) = 0$$

$$\Rightarrow \quad x-3 = 0 \quad \text{or} \quad x-7 = 0$$

$$\Rightarrow \quad x = 3 \qquad\qquad x = 7$$

Note: $x \neq 2$,
$x \neq 5/2$

$$\left[\ \frac{x+2}{2x-5} = \frac{2x-1}{x-2}\ \right] \qquad \times(2x-5)(x-2)$$

[or cross multiply]

$$\Rightarrow \quad \frac{(2x-5)(x-2)(x+2)}{2x-5} = \frac{(2x-1)(2x-5)(x-2)}{x-2}$$

$$\Rightarrow \quad (x-2)(x+2) = (2x-1)(2x-5)$$

$$\Rightarrow \quad x^2 - 4 = 4x^2 - 12x + 5$$

$$\Rightarrow \quad 0 = 3x^2 - 12x + 9$$

$$\Rightarrow \quad 0 = x^2 - 4x + 3$$

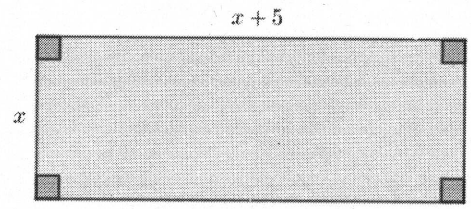
$$\Rightarrow \quad 0 = (x-3)(x-1)$$

$$\Rightarrow \quad x-3 = 0 \quad \text{or} \quad x-1 = 0$$

$$\Rightarrow \quad x = 3 \qquad\qquad x = 1$$

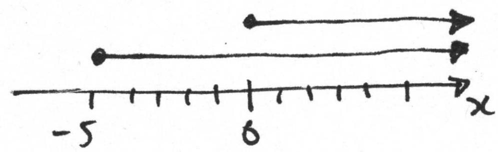
(both permissible)

A rectangle has sides measuring x cm by $(x+5)$ cm. The area of the rectangle is 24 cm^2.



(a) What values of x are valid in this context?

(b) Calculate the lengths of each side.



(a) side lengths must be greater than zero.

$\therefore \quad x > 0 \quad$ and $\quad x + 5 > 0$

$x > -5 \qquad$ both conditions must be satisfied.

Any values of $x > \cancel{-5}0$ will generate a rectangle.

(b) $A(x) = x(x+5)$ length $\times$ width

$= x^2 + 5x; \; x > 0$ [function stated with domain because domain is not $x \in \mathbb{R}$.]

Also, $A(x) = 24$

$\therefore \quad x^2 + 5x = 24$

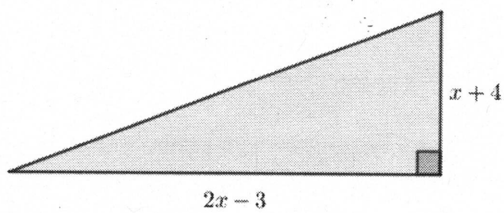
$\Rightarrow x^2 + 5x - 24 = 0$

$\Rightarrow (x-3)(x+8) = 0$

$\Rightarrow x - 3 = 0 \quad x + 8 = 0$

$\boxed{x = 3} \quad \boxed{\cancel{x = -8}}$ ← does not satisfy condition that $x > \cancel{-5}0$, so reject this solution.

A right-angled triangle has shorter sides measuring $(2x - 3)$ cm and $(x + 4)$ cm, as shown in the figure.



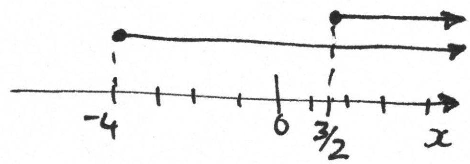
(a) What values of x are valid in this context?
(b) Calculate $A(x)$, the area of the triangle in terms of x. State the domain.
(c) At what value of x is $A(x) = 20\text{ cm}^2$?
(d) Show that the hypotenuse h is given by the function
$h(x) = \sqrt{(5x^2 - 4x + 25)}$ cm.
(e) Calculate x when $h(x) = \sqrt{37}$.

(a) $2x - 3 > 0$ <u>and</u> $x + 4 > 0$

$x > -4$

$2x > 3$

$x > 3/2$



To satisfy $x > 3/2$ <u>and</u> $x > -4$, <u>$x > 3/2$</u>.

(b) Area triangle = $\frac{1}{2}$(base)(height)

$A(x) = \frac{1}{2}(2x-3)(x+4)$

$= \frac{1}{2}(2x^2 + 5x - 12)$

$= x^2 + \frac{5}{2}x - 6$, $x > 3/2$ ← domain

(c) $\left[20 = x^2 + \frac{5}{2}x - 6\right] \times 2$

$\Rightarrow$ $40 = 2x^2 + 5x - 12$

$\Rightarrow$ $0 = 2x^2 + 5x - 52$

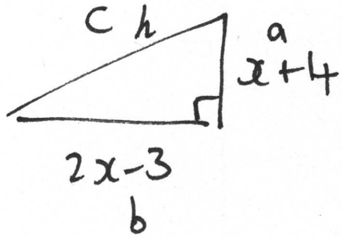
$\Rightarrow$ $0 = (2x + 13)(x - 4)$

$\Rightarrow$ $0 = 2x + 13$ or $x - 4 = 0$ $\Rightarrow$ $\boxed{\cancel{x = -13/2}}$ or $\boxed{x = 4}$ ✓

reject because $x > 3/2$

$\left(\begin{aligned} 52 &= 1 \times 52 \\ &= 2 \times 26 \\ &= 4 \times 13 \end{aligned}\right)$

(d)



$$a^2+b^2=c^2$$

$$[h(x)]^2 = (x+4)^2 + (2x-3)^2$$

$$= (x+4)(x+4) + (2x-3)(2x-3)$$

$$= x^2 + 8x + 16 + 4x^2 - 12x + 9$$

$$= 5x^2 - \overset{4}{\cancel{16}}\,x + \cancel{10}25$$

$$\Rightarrow \quad h(x) = \sqrt{5x^2-4x+25}\,, \quad x > 3/2$$

(e) $h(x) = \sqrt{37}$

$$\sqrt{37} = \sqrt{5x^2-4x+25}$$

$$37 = 5x^2-4x+25$$

$$0 = 5x^2-4x-12$$

$$0 = (5x+6)(x-2)$$

$12 = 1 \times 12$
2×6
3×4

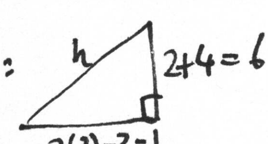
$5x+6=0$ or $x-2=0$

$5x=-6$ $\quad x=2$

$x=-6/5$

Reject $x=-6/5$ as this does not satisfy condition that $x>3/2$.

$x=2$ gives hypotenuse $\sqrt{37}$. Check:



$6^2+1^2=37$ ✓